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**APPLICATION OF RELIABILITY ANALYSIS TO  
AIRCRAFT STRUCTURES SUBJECT TO  
FATIGUE CRACK GROWTH AND  
PERIODIC STRUCTURAL INSPECTION**

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*BOEING COMMERCIAL AIRPLANE COMPANY*

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## FOREWORD

The research work reported herein was conducted by the Boeing Commercial Airplane Company for the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under Contract F33615-71-C-1134, Amendment No. P00002. This contract was initiated under project 7351, "Metallic Materials," task 735106, "Behavior of Metals," with Mr. R. C. Donat acting as project engineer.

The study was conducted at the Boeing Commercial Airplane Company, Structures Technology Staff, Fatigue Research Group, in Renton, Washington, under the direction of Mr. J. P. Butler as program manager. The period covered by this effort is July 16, 1972 through March 31, 1973, and the report was completed in March 1973.

The research was conducted by Mr. Ian C. Whittaker of the Fatigue Research Unit of the Boeing Commercial Airplane Company with support in the statistical developments by Prof. Sam C. Saunders of the Department of Mathematics, Washington State University.\* Acknowledgement is due Mr. Richard E. Clemmons of the Boeing Computer Services, Inc., for necessary computer programming support.

This technical report has been reviewed and is approved.

A handwritten signature in black ink, appearing to read "W. J. Trapp", with a long horizontal flourish extending to the right.

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## ABSTRACT

A method of simulating crack growth has been investigated. The proposed model, which is based on linear elastic fracture mechanics theory, allows for the variability in crack growth behavior found in the experimental data of various materials. Given a reference stress intensity factor range and central tendency values for the crack growth rate and the exponent of the stress intensity factor excursions of a material in a specified configuration, Monte Carlo simulation is used to select various combinations of parameters. These are then used to generate fatigue cracks, on the assumption that crack growth rate is a power function of the stress intensity factor range. The residual strength of the cracked structure is considered to be a decreasing function of the induced crack length. The probability of crack detection also depends on the generated crack and is assumed to improve with increasing crack length. However, this improved detection probability is modified by the probability that the crack location is not the one being inspected. These developments have been used to update the reliability analysis system defined in an earlier report (AFML-TR-72-283). Results of an application of the modified reliability analysis system to a hypothetical, but plausible, situation are presented.

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## LIST OF ABBREVIATIONS AND SYMBOLS

### ABBREVIATIONS

E	mathematical expectation
exp	exponential function
log	common logarithm
ln	natural logarithm

### SYMBOLS

b	exponent in stress intensity factor equation
c	intercept in stress intensity factor equation
K	stress intensity factor
$q_c$	probability of not detecting a crack during a cursory inspection
$q_s$	probability of not detecting a crack during a strict scheduled inspection
s	total length of crack
$s_0$	the initial length of the fatigue crack which occurs at characteristic time $\beta_0$
$T_0$	the actual crack initiation time, to length $s_0$ , in the individual structure
$\alpha_0$	shape parameter of Weibull distributed crack initiation times
$\beta_0$	characteristic life of Weibull distributed crack initiation times
$\gamma$	parameter reflecting structural configuration and stress level
$\delta$	residual strength of a structure containing a fail-safe sized crack
$\eta$	crack growth rate
$\theta$	inspection thoroughness parameter
$\kappa$	common logarithm of reference stress intensity factor
$\lambda$	common logarithm of crack growth rate at reference stress intensity factor
$\mu_1$	mean value of the crack growth rate exponent
$\nu$	number of equal inspection sampling subsets in a fleet
$\sigma_1^2$	variance of crack growth rate exponent
$\sigma_2^2$	variance of $\lambda$
$\tau$	threshold length for crack detection
$v$	fraction of details in a structural component which are examined during inspection

## SECTION I

### INTRODUCTION

The integrity of aircraft structures has been considered, in past years, to be a function of the designed or demonstrated static strength. In recent years, however, the emphasis has changed as a result of the increased operational lifetimes demanded of airplane structural systems. This has resulted in the broadening of the strength design concept to include, initially, fatigue evaluation techniques and, most recently, damage tolerance methods as part of the structural integrity task.

The application of these differing technologies often results in inconsistent requirements which are resolved by some fairly arbitrary procedures. To rationalize the impact of these structural integrity requirements on structural design, a study was accomplished on the interaction of the basic variables of static strength, fatigue damage initiation, fatigue crack growth, the fatigue-cracked residual strength, and the coincident environmental load exposure. The resultant reliability analysis scheme, fully described in Reference 1, provides a basic methodology to evaluate or weigh the variables that comprise a modern aircraft structure. The ultimate goal is to ensure structural design that is sufficiently strong, very durable, and resistant to unexpected fatigue damage.

Complicating elements in the development of the reliability procedure involved the modeling of the material and structural characteristics in the presence of a fatigue crack, and the operational procedures for inspecting the structure for possible damage. For example, ultimate strength is lowered by more than that implied by the net area reduction of the actual cracked structure. Furthermore, both material type and structural geometry affect the strength response to a fatigue crack. Closely related to this strength variable is the rate at which the crack grows in the structure, under the influence of the operational loads, to the critical or designed size limit.

The rate of crack propagation also has a direct bearing on the likelihood of detecting the crack prior to its reaching a critical size. Accordingly, a most complex interaction of material characteristics, structural design configuration, operational load exposure, and operational inspection procedures defines the condition and reliability of a structure at some time after crack initiation.

The following section of this report presents a refinement of the reliability analysis developed in Reference 1. The provisional crack growth, residual strength, and crack detection models presented in that report have been redefined in this section to include concepts from linear elastic fracture mechanics.

Section III discusses the results of some exploratory parametric studies using the improved reliability system. These studies were based on a hypothetical case similar to the one used in Reference 1.

Section IV presents the conclusions of this exploratory study.



## SECTION II

### ANALYTICAL DEVELOPMENT OF CRACK GROWTH, RESIDUAL STRENGTH, AND CRACK DETECTION MODELS

A structural reliability model has been developed, Reference 1, which presumes that at some time after the introduction of an airplane into service, a fatigue crack is likely to initiate in some critical piece of structure, due to the regular imposition of a loading spectrum through service usage. The crack will propagate at a rate dependent on the material, structural geometry, and applied loads, until either it is detected during an inspection, or it reaches a length when it is arrested by some design constraint, or the structure fails. It is further assumed that the strength of the structure will diminish as the crack grows, and vice versa, the chances of detection will improve with the increase in crack size.

The dependency of the residual strength and inspection functions on the actual crack length at any given time places considerable emphasis on the crack growth model. Although a complete and exact definition of this material-structural behavior is not available, the application of fracture mechanics principles is gaining widespread acceptance as the basis for predicting crack growth. Furthermore, a considerable amount of cyclic load data has been presented in the literature in the form of stress intensity versus crack growth rate curves. Therefore, it has been decided to incorporate a linear-elastic fracture mechanics approach to compute crack growth in the reliability analysis system of Reference 1 in lieu of the originally assumed  $\xi$ -normal distributional behavior. In this new approach, the fatigue crack growth rate is considered to be a function of the particular stress intensity factor and its variation in the vicinity of the fatigue crack tip, Reference 2. The physical-metallurgical aspects of this approach will not be discussed here, because it is intended only to postulate what is believed to be an appropriate stochastic model for crack behavior as it is presently understood.

#### 1. CRACK GROWTH MODEL

The concept underlying the development of the fatigue crack model is one which involves several dependent phases. A model of crack development and growth must span a material's condition from its "as-received" state to the fatigue-cracked or fractured stage of a part or structure under the variable loads of use. Fatigue crack initiation and detection conventionally provide a real measure or control of the potential fatigue performance of a structure. For practical purposes a crack is said to be initiated whenever it reaches a size which can be observed or measured with the aid of some instrumentation or procedure. The time to initiation will be different for each component of a group, even though nominally identical structures of the same material are subjected to the same external loads. The crack initiation time, as proposed in References 3 through 7, is taken as a random variable with an extreme value distribution and identified as a two-parameter Weibull distribution having a given characteristic life and a defined shape parameter.



This random initiation behavior, as defined above, has proceeded through an incubation phase and a physically recognizable growth to the detected size, regardless of the sensitivity of the detection method. At present when fatigue-cracked components are examined to determine the crack initiation time, current technology provides a means of recognizing initiation sizes in the order of 0.001 to 0.002 inch. However, this is usually possible only under ideal circumstances. Typical airplane structure, when loaded, has a rather complicated stress pattern, and under normal operational conditions a structure is subjected to a varied set of loads. Consequently, any typical structural component which has been operated has a very complex stress history. Therefore, when fatigue-critical structures are examined to determine condition, the cracks are usually detected at initiation sizes only in the order of 0.02 to 0.03 inch, although fractographic examination can sometimes trace origins to lesser size. Crack growth definition in fracture mechanics principles is characterized in terms of the maximum or range of stress intensity and the growth rate. In such a form, crack growth is significantly affected by two threshold levels. At the initiation stage and a low stress intensity level, the crack growth rate is practically independent of the variable stress intensity factor. An upper threshold limit is defined by the maximum critical stress intensity factor which is associated with crack tip growth velocities approaching the speed of sound in the material. In Figure 1, some derived crack growth rate data, Reference 8, are presented. The lower threshold of crack growth rate is quite apparent. Also apparent is a region in which the crack growth rate appears linear on this logarithmic plot. This linear stage extends to a point somewhat short of the critical stress intensity factor, where it exhibits an asymptotic behavior. The usual assumption will be that the crack at initiation is of a size which can normally be detected. Hence, incubation and the related stress intensity level threshold growth behavior phases are effectively overridden by definition of an initiation crack size.

The model for this phase of crack growth, considering post-initiation size, is based on the assumption of the validity of two laws from fracture mechanics. These are that between the limits of the threshold stress intensity factor and the critical stress intensity factor:

- the logarithm of the crack growth rate is a linear function of the logarithm of the stress intensity factor, and
- the stress intensity factor is proportional to the square root of the crack length at any fixed stress level for the specific geometry.

It has been suggested in the literature, Reference 9, that the relationship of crack growth rate,  $\eta$ , for any given stress ratio is of the form:

$$\eta = cK^b$$

where the intercept,  $c$ , and the exponent,  $b$ , are stochastic values. Now if  $K(s)$  denotes the stress intensity factor for a crack of length  $s$ , the above assumptions may be written in the form:

$$\log \eta = \log c + b \log K(s)$$

and

$$K(s) = \gamma \sqrt{s}$$

An examination of some readily available literature, such as References 10 to 14, provides information on typical values for the parameters just discussed. Figures 2 through 5 are examples of such data and are presented to illustrate the variability in the performance of similar specimens. The parameter  $\gamma$  reflects the applied stress and geometrical configuration of the structure and, as the data in each figure represent similar specimens, this parameter may be treated as a constant. The two remaining parameters, namely the exponent,  $b$ , and the intercept,  $c$ , are the ones which reflect the variability demonstrated between tests. It is believed that only the exponent,  $b$ , is an independent random variable. In other words, it is presumed that these parameters basically act as a couple  $(b,c)$  which is random and whose joint distribution must be determined from data such as are shown in Figures 2 through 5, along with some mathematical constraints which will be developed in the following paragraph.

Combining the two fracture mechanics equations of the preceding paragraph and for a given  $(b,c)$ , the crack growth rate for a crack of size  $s$ , when  $t \geq t_1$  is given by:

$$\eta = c(\gamma^2 s)^{b/2}$$

The possible boundary conditions are that the rate of change of the crack growth be nondecreasing and that the crack length be continuous at  $t_1$ . This is stated as:

$$s(t_1) = \omega_1 \tag{1}$$

and

$$\eta(t_1) \geq a_1 \tag{2}$$

where  $t_2$  is the time at which the initiated crack reaches macroproportions. Imposing both (1) and (2) requires that the couple  $(b,c)$  be functionally related to  $a_1$  by the inequality

$$a_2 = c\gamma^b \omega_1^{b/2} \geq a_1 \geq 0 \tag{3}$$

and therefore

$$\eta = a_2(s/\omega_1)^{b/2}$$

It is necessary to determine empirically the stochastic relationship between the couple  $(b,c)$ . Figures 2 through 5 show data in the form:

$$y = bx + d$$

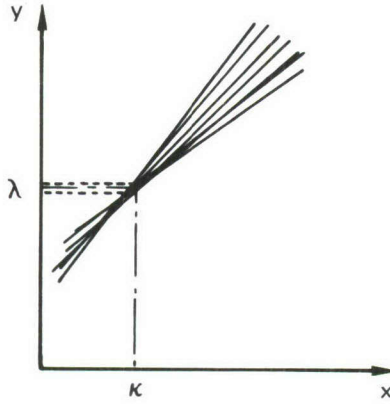
where

$$x = \log K(s)$$

$$y = \log \eta$$

$$d = \log c$$

If  $K_I$  is a material reference stress intensity factor and  $\kappa = \log K_I$ , it is seen that about this reference value there is a variation in the slope, i.e., the exponent  $b$ , and a small variation in the growth rate. This is shown in the schematic which follows:



This joint relationship may be expressed, using capital letters to denote random variables, as

$$y = \Lambda + B(x - \kappa)$$

where  $\Lambda$  and  $B$  are jointly determined once measurements are made on a metallic specimen. Over the population of such specimens these values would have a distribution which is presently unknown. Presuming this to be the most convenient formulation of this stochastic variability, they are taken to be normal random variables

$$B \sim N(\mu_1, \sigma_1^2)$$

and

$$\Lambda \sim N(\lambda, \sigma_2^2)$$

Imposing the constraints of the preceding paragraphs, namely

$$C = 10^{\Lambda} / K_I^B$$



and the equality in equation (3)

$$\log C + B(\log \gamma + \log \sqrt{\omega_1}) = \log A_1$$

it is also necessary to have the equation relating random variables

$$\Lambda + B[\log (\gamma/K_I) + \log \sqrt{\omega_1}] = \log A_1$$

This imposes conditions on the parameters

$$\lambda + \mu_1 [\log (\gamma/K_I) + \log \sqrt{\omega_1}] = m$$

for the means, and

$$\sigma_2^2 + \sigma_1^2 [\log (\gamma/K_I) + \log \sqrt{\omega_1}]^2 = \xi^2$$

for the variances.

To summarize, therefore, crack growth in a structure is assumed to be a multistage process. The first stage involves the accumulation of fatigue damage to the time of initiation of a small crack. Next follows the initial growth of the crack from its initiated microsize to macroproportions, at which point the third stage of crack growth is assumed to begin. For almost all practical applications the second stage of the crack growth model will be redundant as the initiation size of a crack in a structure will be of macroproportions. Consequently, the normal procedure will be to compute the time to initiation of a crack of detectable size and then compute the growth of the crack per the third stage crack growth model. This third stage of crack growth is assumed to follow fracture mechanics principles, namely, the rate at which the crack grows is a function of the stress intensity factor and its variation at the crack.

## 2. RESIDUAL STRENGTH MODEL

The incorporation of fracture mechanics concepts, to compute crack growth in the structural reliability system, facilitates the redefinition of the tentative residual strength model of Reference 1 to reflect this same technology. It is presumed that the strength of a structure, or structural component, remains constant until the initiation of a fatigue crack. As the crack propagates, the residual strength of the cracked article diminishes until it is insufficient to sustain the applied load and the structure fractures. However, current airplane structural design technology emphasizes the fail-safe approach, where materials with slow growth rates and good fracture properties are often used in conjunction with positive crack stoppers, which provide the means of arresting the cracks at some predetermined fail-safe length,  $s_1$ . Thus, the fail-safe crack length defines the lower limit of the residual strength of a seriously cracked structure. Furthermore, as this fail-safe residual strength is a design constraint, it is usually substantiated by test and/or analysis and is, therefore, normally available as an input parameter.

Now for any given structure there is a critical value of the stress intensity factor, usually labelled  $K_c$  for plane stress conditions, which defines the critical crack length,  $s_c$  for a specified loading condition. This is the length at which the crack becomes unstable and will grow with great rapidity until it is either arrested or the structure fractures. The critical crack length is essentially a material parameter and, therefore, is not necessarily the same as the structural fail-safe length. Consequently, in those cases where the critical and fail-safe crack lengths differ, i.e.,  $s_c \neq s_1$ , it is assumed that the lower limit of the residual strength of the structure coincides with the minimum of either the critical or the fail-safe crack length.

Based on these considerations, the following residual strength model is proposed.

Let  $L(s)$  be the residual strength of some structure containing a fatigue crack of length  $s$ . The original strength of the structure is assumed to remain constant until the initiation of a crack of length  $s_0$ . Therefore, for  $s \leq s_0$

$$L(s) = \delta_u$$

where  $\delta_u$  is the ultimate strength design parameter, and

$$L \left\{ \min (s_c, s_1) \right\} = \delta$$

where  $\delta$  is the fail-safe residual strength design parameter. These strength design parameters are normally related to the design limit strength of the structure,  $\delta_\ell$ , as follows:

$$\delta \leq \delta_\ell < \delta_u$$

Furthermore, on current fail-safe airplane structures, typically:

$$\delta_u = 1.5\delta_\ell$$

and,

$$\delta > 0$$

Taking into account the foregoing definitions, the residual strength of a structure or structural component containing a fatigue crack of length  $s$  is given by

$$L(s) = \delta_u - [K(s)/K_c] [\delta_u - \delta]; s > s_0$$

Where  $K(s)$ , as explained earlier, is the stress intensity factor of the particular structure under consideration at that point in time when the length of the fatigue crack is  $s$ .

### 3. CRACK DETECTION MODEL

So far the discussion has dwelt on the initiation of fatigue damage in a structure subjected to cyclic loads, the propagation of this damage in the form of a crack of



monotonically increasing length, and the simultaneous degradation of the strength of the structure as a result of this growing crack.

However, normal operational procedure requires the periodic examination of aircraft structure, which helps maintain the integrity of the structure, as detection of damage is followed by corrective action and renewal of the structural strength. Without inspections, it is anticipated that the reliability of an airplane fleet, as a function of time, would decrease toward zero, instead of maintaining a level of approximately unity as in the case of an inspected, renewable fleet.

The length of the fatigue crack, which was seen to be a major parameter in the strength degradation model, is also assumed to exert considerable influence on the crack detection model. However, unlike the residual strength case, the probability of detection is taken to be an increasing function of the crack size. Unfortunately this is not the only parameter for consideration in a crack detection model, as it must also recognize that the size of a fatigue crack is immaterial if the correct, i.e., cracked, location is not the one that is being inspected.

Therefore, the structural inspection model proposed here assumes that the probability of detecting a crack in a structure depends first on examining the correct location and subsequently on the resolution capability of the crack detection method used for the inspection. Furthermore, this proposed model distinguishes between the regularly scheduled, strict inspection and the more superficial surveillance that occurs either during maintenance tasks in the immediate neighborhood of the crack or from the day-to-day walkaround inspections. Provision is also made so that distinction can be made between the differing structural configurations usually inspected. It is presumed that a crack in a structural component comprising but a single detail, such as a lug, will be detected with more certainty during an inspection than a crack in a detail of a component comprising a multiplicity of similar details, such as the fastener locations of a splice.

Finally, some additional assumptions are necessary regarding the latter case of the multidetail structural component. Namely, it is assumed that the probability of crack detection is also dependent on the ratio of the number of details examined to the total number of details and the lifetime of the component when it is inspected. It is quite evident that if all the details are examined the probability of crack detection is higher than it would be when only small fractions of the details are looked at during each inspection period. It is suggested that early in the lifetime of the structural component none, or at most one, of the details would contain a crack of detectable size. However, as the component ages it is expected that an increasing number of details would contain such cracks. It is obvious, therefore, that even without considering the increasing crack size, the probability of detecting a crack is improved when the structural inspection is performed late in the life of the component, and especially so in the case when only a percentage of the details is examined at any single inspection period.

Although the discussion thus far has dwelt on the probability of crack detection, structural reliability is more easily expressed as a function of the probability of *not* detecting an existing crack. The ensuing development, therefore, is geared to this approach.



At any time during the life of a structure or structural component, if there exists a fatigue crack of size  $s$ , then  $q(s)$  is taken to be the probability of not detecting the crack. This probability level depends on the probabilities

- (1)  $q_d$ , of not inspecting the correct, i.e., cracked detail, and
- (2)  $q_r$ , of not being able to detect the crack, given that the correct detail is the subject of the inspection.

It is seen that condition (1) must be a function of the total number of similar details in a structural component, of which any or all could be fatigue cracked, and the number of details actually examined during an inspection. On the other hand, condition (2) is simply a function of the resolving capability, under normal operation, of the crack detection technique used for the inspection.

Considering the condition (1) case first, let

$M$  = the total number of similar details in the structure

$m$  = the number of details examined during an inspection period

Now at some time  $t$ , none, one, or more than one detail could contain a fatigue crack of detectable size. This is directly related to the assumptions made regarding time to initiation of the first crack in the structure or component, for it is obvious that in order for the structure to be cracked at least one detail must be cracked. Considerably earlier in the discussion it was stated that the assumed time to crack initiation,  $T_0$  can be represented by the Weibull distribution with parameters  $(\beta_0, \alpha_0)$ . Now, time to initiation of the first crack in a structure or component, comprising  $M$  details, and time to crack initiation in the weakest detail are obviously one and the same. Therefore, it is seen that the structure's scale parameter,  $\beta_0$ , actually represents the characteristic life of the weakest details within several similar structures. Consequently, it is assumed that the crack initiation times,  $T_d$ , of the details are also Weibull distributed but with a different scale parameter,  $\beta_d \geq \beta_0$  depending on  $M$ , which is given by:

$$\beta_d = \beta_0 M^{1/\alpha_0}$$

Now if  $T_1, \dots, T_M$  are the times to initiation of fatigue cracks in the  $M$  details of the structure, then the number of uncracked details at any time,  $t > 0$ , is  $U(t)$ , where

$$U(t) = \sum_{i=1}^M \{T_i > t\}$$

where  $\{T_i > t\}$  is one if the relation is true and zero otherwise. Therefore, in the general case when  $m$  observations are made from the total of  $M$  details, in which cracks appear at random times  $T_i$  for  $i = 1, \dots, M$ , the probability of not selecting a cracked detail is

$$\begin{aligned}
q_d &= \frac{U(t)}{M} \cdot \frac{U(t)-1}{M-1} \cdots \frac{U(t)-m+1}{M-m+1} \\
&= \frac{\binom{U(t)}{m}}{\binom{M}{m}}
\end{aligned}$$

Now letting  $V(t) = M - U(t)$  be the number of cracked details, then it may be stated that at any time  $t > 0$

$$q_d(t) = \left(1 - \frac{V(t)}{M}\right) \left(1 - \frac{V(t)}{M-1}\right) \cdots \left(1 - \frac{V(t)}{M-m+1}\right)$$

If  $M$  is large and  $t$  such that  $V(t)$  is small, which is the expected situation under discussion, then

$$q_d(t) \simeq \left(1 - \frac{V(t)}{M}\right)^m = \left(1 - \frac{V(t)}{M}\right)^{Mv} \simeq e^{-vF(t)}$$

where  $v = m/M$ , the fraction of the details which are inspected, and  $F$  is the Weibull law by which  $T_1, \dots, T_M$  are identically distributed.

The approximation follows, since as  $M \rightarrow \infty$  it is known that

$$V(t) \rightarrow F(t) \quad \text{for } t > 0$$

So much for condition (1). Consider now the case defined by condition (2), which simply reflects the resolution or sensitivity of the crack detection system employed for the inspection. Presuming that the correct location is being examined, it is obvious that a crack of larger size will be detected more easily than a smaller one. Therefore, the proposed detection model will be basically crack-size dependent. More specifically, it is assumed to be a function of the largest existing fatigue crack, in the structure or component, at the time of the inspection. An additional size parameter must also be considered to reflect the threshold in the sensitivity of the crack detection device. This threshold will vary according to the type of device employed, and the operational and physical constraints imposed on the inspection procedure. Finally, a detection quality parameter is required to distinguish between the different levels of structural inspection which occur during the operational lifetime of a fleet. For example, at scheduled periods some of the various components may be examined in their fully assembled condition, whereas others may be inspected after being stripped down. Obviously the latter will be more thorough and, therefore, this will be represented by a suitably better value for the detection quality parameter. On the basis of these assumptions, it is proposed that the probability of not detecting a crack in a structure, given that one of size  $s$  exists in the detail being examined, is given by

$$q_r(s) = \exp[-\theta (s - \tau)^+] \quad \text{for } s > 0$$



where

$\theta$  = inspection thoroughness parameter

$\tau$  = detection threshold length

$$(s - \tau)^+ = \begin{cases} s - \tau & \text{if } s > \tau \\ 0 & \text{otherwise} \end{cases}$$

Now that the terms  $q_d$  and  $q_r$  have been defined it is possible to evaluate the probability of not detecting a fatigue crack of length  $s$  in a structure during an inspection period. If this probability is  $q_s(s)$ , then it has been argued that this is a function of both  $q_d$  and  $q_r$ .

As there are these two sources by which a crack in a structure may not be discerned, the total probability of detecting a crack is decreased. The probability of not detecting a crack equals the probability of inspecting a cracked detail and not detecting a crack plus the probability of inspecting only uncracked details. Therefore, the probability that a crack in a structure is not detected at an inspection period is given by

$$q_s = (1 - q_d)q_r + q_d = q_d + q_r - q_r q_d$$

The crack detection model, as discussed so far, has been oriented toward the scheduled structural inspection, hence the label  $q_s$ . During normal operational service these strict inspections are reinforced by superficial examinations during the course of maintenance tasks. For obvious reasons this category of inspection usually applies only to exposed portions of a structure, which are easily visible. However, some less accessible structure may also be subjected to these cursory checks when the maintenance task requires some "tear-down," thereby exposing the hidden structure. Cursory inspections, as defined here, are usually visual scanings of the structure and so would normally encompass the entire area of interest. Consequently, in the case of a multidetail structure or component, the concept of making  $m$  observations from a total of  $M$  details is not too meaningful as it is more likely that all  $M$  details would get screened. Moreover, the probability of detecting a crack under these conditions is likely to be almost totally dependent on the size of the largest crack in the structure and relatively insensitive to the possibility of other smaller cracks also being present.

The cursory inspection model proposed here is similar, therefore, to the  $q_r$  model previously described, namely, that the probability of not detecting a crack of length  $s$  during cursory inspection is given by

$$q_c(s) = \exp \left[ -\theta_c (s - \tau_c)^+ \right]$$

In this usage  $\tau_c$  is again a crack detection threshold length, but of rather grosser dimensions than  $\tau$ ; and  $\theta_c$  is a parameter which reflects the degree of difficulty for visual scanning of the structure. A major factor influencing this parameter will be the magnitude of the structure being surveyed, since it is easier to miss a crack when scanning a large area rather than a small one.



Based on all the preceding arguments and assumptions, it follows that given a structure which contains a crack of length  $s$  and is subjected to both scheduled and cursory inspections, the probability of not detecting the crack is given by

$$q(s) = q_c q_s$$

One more aspect of the inspection process must be considered. This involves the standard operational procedure of scheduling only some fraction of the fleet for structural examination at any single inspection period. In order to maintain simplicity and facilitate development, it is assumed that the inspected portions of the fleet are of equal size. In other words, it is assumed that  $\nu$  subsets, each containing  $n$  units, comprise a fleet of  $W$  airplanes, or structural components, which is the total exposure.

At the first inspection period the structures of the airplanes in the first group, i.e., subset  $N_1 = (1, 2, 3, \dots, n)$ , are examined. At the  $j^{\text{th}}$  inspection period, (where  $j = 1, 2, 3, \dots, \nu$ ), the airplanes in the subset  $N_j = [(j-1)n + 1, (j-1)n + 2, \dots, jn]$  are examined and at the  $(\nu + 1)$  inspection the subset  $N_1$  is again inspected. This may be restated more concisely as follows: at the  $k^{\text{th}}$  inspection period, for  $k = 1, 2, \dots$ , the subset  $N_j$  is examined iff  $k = j \bmod \nu$ , that is, if there exists an integer,  $i$ , such that  $k - j = i\nu$ .

Now if  $s_j(t)$  is the length of the fatigue crack in the  $j^{\text{th}}$  structure at some time  $t > 0$ , then for each  $j = 1, \dots, W$  the observed  $s_j$  is a nondecreasing stochastic process. Hence,  $s_j(k)$  is the length of the crack at the time of the  $k^{\text{th}}$  inspection for  $k = 1, 2, \dots$ . Therefore, it follows that the probability that a crack of length  $x \geq 0$  is undetected during the  $k^{\text{th}}$  inspection period is  $q_{kj}(x)$ . So for  $j = 1, \dots, W$  and  $s \geq 0$

$$q_{ij}(x) = \begin{cases} q(x) & \text{iff } j \in N_k \\ 1 & \text{otherwise} \end{cases}$$

where  $i = k \bmod \nu$ .

Thus,  $q_{kj}[s_j(k)]$  is the probability that the fatigue crack in the  $j^{\text{th}}$  structure at the time of the  $k^{\text{th}}$  inspection and of length  $s_j(k)$  is undetected.

#### 4. APPLICATION TO RELIABILITY MODEL

It is apparent that the relationship between two of the parameters mentioned in the preceding discussion requires some additional comment. These parameters, the crack initiation size and the crack detection threshold length, are independent of one another, each arising from a differing set of physical constraints. For example, current fatigue life prediction technology is heavily dependent on experimental verification. These supporting data are frequently obtained from tests on large specimens several feet long, or the complete structural component, or at times the full-scale structure. The typical size of cracks which are found and repaired on such tests is in the neighborhood of one-tenth of an inch. As a result, fatigue life predictions which are based on such data are actually estimates of the central tendency times to initiation of similarly sized cracks in similar structure but under

operational conditions. This estimate of average time to crack initiation is used in the reliability analysis as the scale parameter  $\beta_0$ , which defines the characteristic time,  $T_0$ , to initiation of a crack of length  $s_0$ .

It has also been stated earlier that the reliability model recognizes a threshold crack size which is a function of the crack detection technique used for structural inspection. This threshold size,  $\tau$ , defines the minimum detectable crack size and is independent of the length  $s_0$ . Consequently, in those instances when  $\tau < s_0$ , which will probably be the general case, there will be some small probability that a crack is detected before the estimated time  $T_0$ , when the crack is of length  $s_0$ . This means that the random variable,  $T_0'$ , when the fatigue crack is of size  $\tau$ , and where  $0 \leq T_0' < T_0$  (for  $\tau < s_0$ ), must be considered as the initial time when a fatigue crack can be detected. The proposed structural reliability model, therefore, recognizes this variable  $T_0'$ , and it is from this time on that the probability of crack detection increases as the crack propagates and the residual strength of the structure begins to diminish.

It should be emphasized that the preceding discussion, together with those on crack growth, residual strength, and crack detection, are modifications and refinements to the original provisional models described in Reference 1. The exploratory studies which will be discussed in the following section have been based, therefore, on a reliability analysis system which incorporates the new procedures defined on the preceding pages but is otherwise identical to the model described in Reference 1.



## SECTION III

### DISCUSSION OF EXPLORATORY APPLICATION AND RESULTS

The results of parametric studies using the proposed crack growth and detection models are discussed in this section. First, the proposed crack growth model will be substantiated. Some typical crack growth rate versus stress intensity factor data, on two common aluminum alloys, a titanium alloy, and a steel, are presented in Figures 2 through 5. These figures show least square fits of the analytical results, obtained using fracture mechanics principles, of experimental crack growth versus cyclic life data. Analyzed results such as those shown in these four figures have been examined and the reduced information is presented in Table 1. Central tendency values for the parameters describing crack growth in 2024-T3 bare sheet were selected from the data in Table 1 and Figure 2. Using the selected parameters in conjunction with Monte Carlo techniques, values for the exponent,  $b$ , and intercept,  $c$ , were generated. Based on these values, cracks were propagated so that simulated crack growth versus cyclic life data were developed. Then they were analyzed using fracture mechanics, and crack growth rates versus stress intensity factors determined. Best linear fits of some of these results are illustrated in Figure 6. Figure 7 shows a single set of generated data and the maximum and minimum values obtained from 2000 such data simulations. A comparison of these generated results, with the test data of Figure 2, shows that the suggested crack growth model is quite realistic. The bounds from the 2000 random simulations are seen to neatly encompass the real 2024-T3 bare aluminum sheet data, and the sample selection presented in Figure 6 is almost a carbon copy of the experimental data of Figure 2.

The task of determining the typical values of the growth rate, slope, and variability parameters for several commonly used materials has not been undertaken. Indeed, such a task is of considerable magnitude and beyond the scope of this report. The few preliminary studies conducted to establish the feasibility of the proposed crack growth model, and tabulated in Table 1, suggest that within the range of interest the slope parameter is independent of, or only slightly dependent on, material type. Values for the exponent in the range 3 to 4.5 were observed to be fairly typical of the aluminum, titanium, and steel data examined. The growth rate parameter, however, does appear to be more material dependent, and fairly large variations were observed even with the limited amount of data examined. Although these initial trends have already been noted, more work is obviously needed to adequately define suitable values for these crack growth parameters.

Now that the applicability of the crack growth procedure has been demonstrated, the crack-size-dependent models for residual strength and crack detection can be discussed. Several parametric studies, based on a hypothetical case, were made and the results plotted in Figures 8 through 16 to illustrate the influence of the various parameters which comprise crack detection and residual strength. The example assumes a fleet of 300 structures or structural components. One-quarter of the fleet, i.e., 75 structures, are inspected at each of the inspection periods which are scheduled at 7500 hour intervals. The characteristic life to a fatigue crack of 0.1 inch in length in a critical detail in the structure is taken to be 120,000 hours. The minimum or threshold size for crack detection is assumed at 0.02 inch, and the maximum distance between positive crack stoppers is 14.0 inches. The fail-safe



*Table I.—Crack Growth Variation Observed In Experimental Data*

Thickness, (in.)	2024-T3												7075-T6												Ti-8Al-1Mo-IV Duplex Anneal						AM 350 SCT																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																															
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	L	T	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L

residual strength of this structure is given as 80% of the limit load capability. Finally, the structure is presumed to be an aluminum one with crack growth parameters already defined in Figure 6 and with a critical stress intensity factor,  $K_{Ic}$ , of 75 ksi  $\sqrt{\text{in.}}$

Figure 8 shows an example of the propagating crack and residual strength values given by a single random simulation based on the hypothetical parameters just defined. The example presented in this figure was one of the more extreme or early crack initiation time cases, selected from the very many simulations performed at each computation of structural reliability. It is easily seen that time to initiation of a crack of detectable size can be low in spite of the initial assumption of the 120,000 hour characteristic life. This figure also helps illustrate the time-dependent nature of both crack length and residual strength, with the former an increasing function and the latter a decreasing one.

The impact of the crack detection term on fleet reliability can be seen from Figure 9. The solid line represents the reliability of the fleet of 300 structures or structural components subjected to scheduled inspections, at intervals of 7500 hours, and repair of any detected damaged structures. The chain-dotted line is the probability that either no crack has been detected or no structure has failed in the fleet. It is notable how rapidly this latter curve decreases toward zero, whereas the solid line for the renewed fleet decreases only slightly and quite gradually.

It should be noted that the normal probability scale, against which fleet reliability has been plotted in this and the ensuing figures, was used for reasons of convenience only.

Figure 10 compares the effect of the inspection thoroughness parameter,  $\theta$ , on fleet reliability. As expected, this parameter does not influence fleet reliability initially as the structures are still in the crack initiation stage. However, once cracks of detectable size begin propagating, the  $\theta$  parameter is seen to affect reliability, so that decreasing values of  $\theta$  result in reduced fleet reliability.

Figure 11 also compares the effect of the  $\theta$  parameter except that crack initiation times were computed using a shape parameter,  $\alpha_0$ , of 3.0, whereas in Figure 10 a value of  $\alpha_0 = 4.0$  was used. A comparison of Figures 10 and 11 reveals the similarity in the trend of the results. However, it is obvious that in Figure 11 the inspection thoroughness parameter,  $\theta$ , becomes effective at an earlier time than it does in Figure 10. This follows because of the increased scatter, in crack initiation times, resulting from the imposition of a lower value for the shape parameter of the distribution of times to initiation of fatigue cracks in the structures.

Figure 12 illustrates the improvement in fleet reliability obtained by the addition of cursory inspections. It is assumed, as before, that scheduled inspections occur at 7500 hour intervals and that one-quarter of the fleet is examined at each inspection period. The solid line in Figure 12 represents the case of a hidden or buried detail which is subject to the inspection procedure just outlined. However, the improved chain-dotted line reflects the case of a visible detail which is located in the immediate vicinity of a routine maintenance item. During these maintenance tasks, which in this example occur at 3750 hour intervals, there is a superficial inspection of the surrounding structure. It is assumed that as a result of these visual examinations there is an increased possibility of detecting fatigue cracks which equal or exceed 0.25 inch in length.



Figure 13 is presented to show the impact on fleet reliability of multiple-detail structures or structural components which are subjected to limited inspections. The solid line represents the case of a single detail component which is inspected at an inspection period. The chain-dotted and dashed lines represent a structure comprising ten identical, independent details. It is assumed that for some reason it is only possible to examine one of the details, the dashed line, or five of the details, the chain-dotted line, at any given inspection period. It is obvious from the figure that the latter cases result in a lowered level of fleet reliability. This is only as expected considering that the probability of crack detection decreases with reduced inspection sampling.

Figures 14 through 16 also illustrate the case of the multiple-detail structure, of which only a single detail is inspected at any scheduled inspection period. Figure 14 assumes the case of the deeply buried multiple-detail structure where only a single detail can be inspected at any one scheduled inspection period. A comparison of the results plotted in this figure with those shown in Figure 11 illustrates the extent of the decrease in fleet reliability which occurs from such limited inspection sampling.

In Figure 15 it was assumed that the multiple-detail structure was more accessible. Therefore, although it was still only practical to rigidly inspect a single detail at any given inspection period, it was also possible to superficially inspect the remaining details at the same time. It is immediately obvious from this figure that fleet reliability has improved under this method of rigorously inspecting some of the details and cursorily examining the remainder.

Figure 16 is basically the same case which was presented in the preceding paragraph, except that the structure was considered to be an easily visible one. As such, it was assumed to be the subject of an additional cursory examination, between the scheduled inspection periods, when routine maintenance tasks were being performed in the same vicinity. A comparison of Figure 16 with Figure 15 clearly illustrates the improvement in fleet reliability which results from the additional structural surveillance. In fact, these results are comparable with those from the reference single detail case shown in Figure 11.



## SECTION IV

### CONCLUSIONS

The reliability model developed in Reference 1 was responsive to the material and structural characteristics of fatigue crack initiation, fatigue crack growth, and residual strength, as well as to the operational procedure of periodic inspection, detection, and repair of any cracked structures. This reference incorporated provisional models describing these physical aspects of the proposed reliability analysis system.

The task concluded here involved the development or refinement of those tentative models and their replacement, with the modified models, in the structural reliability system. The areas in which the reliability analysis of Reference 1 was altered are:

- The crack growth function, which now is based on fracture mechanics concepts
- The residual strength function, which now also uses fracture mechanics
- The crack detection function, which now recognizes that the probability of detecting a crack depends first on looking at the correct location and subsequently on the crack size

Parametric studies conducted with these modifications incorporated into the reliability system have shown that

- The crack growth model can provide a good simulation of known experimental data
- Crack length and residual strength are both time dependent
- Fleet reliability is considerably influenced by the crack detection parameters

It is concluded that these modifications have resulted in a workable reliability analysis system, which may be applied to the rational evaluation of the considerable number of variables which make up an airplane structure. However, in reducing this system to practice, additional work is recommended in the definition of typical values for several material, structural, and inspection parameters.

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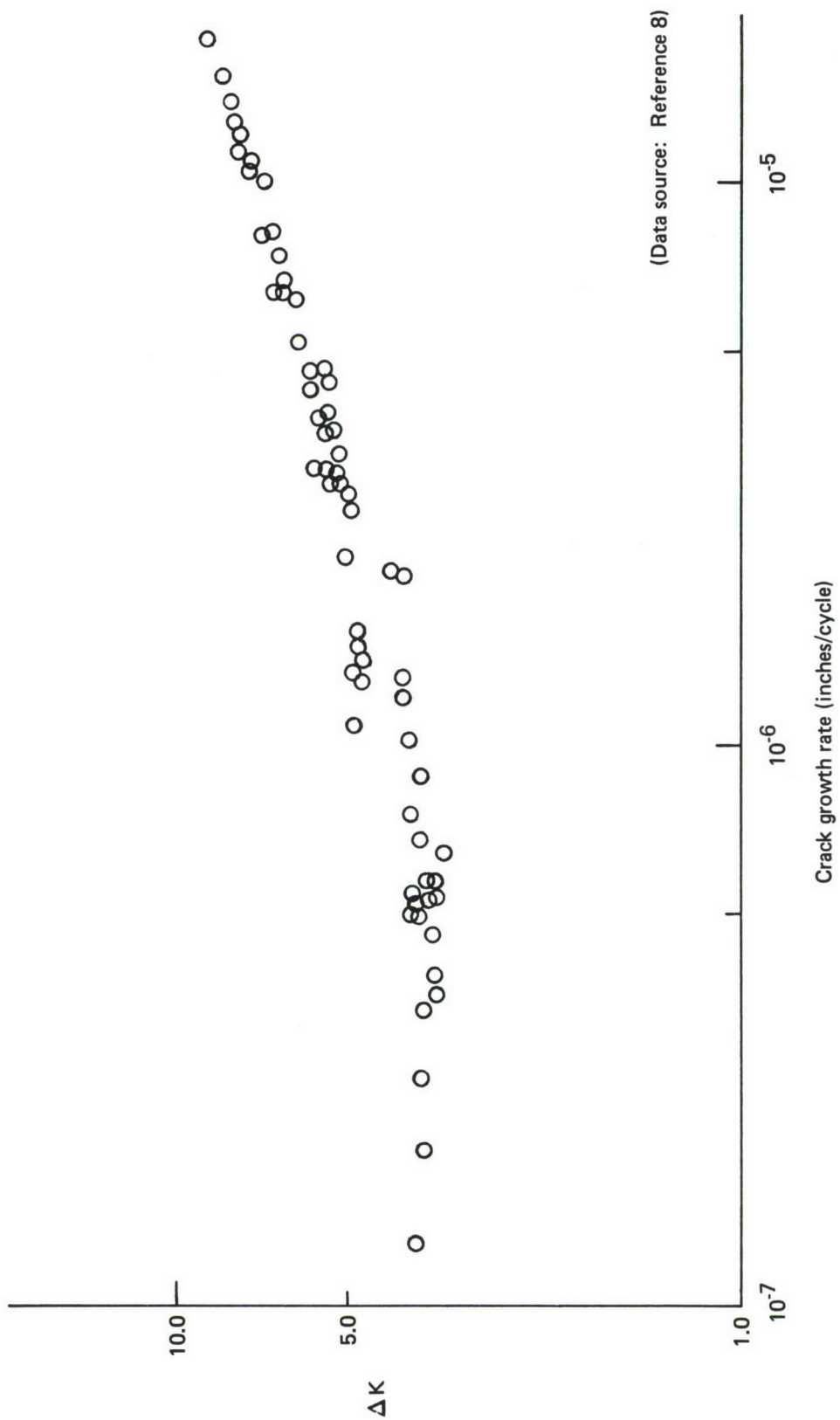
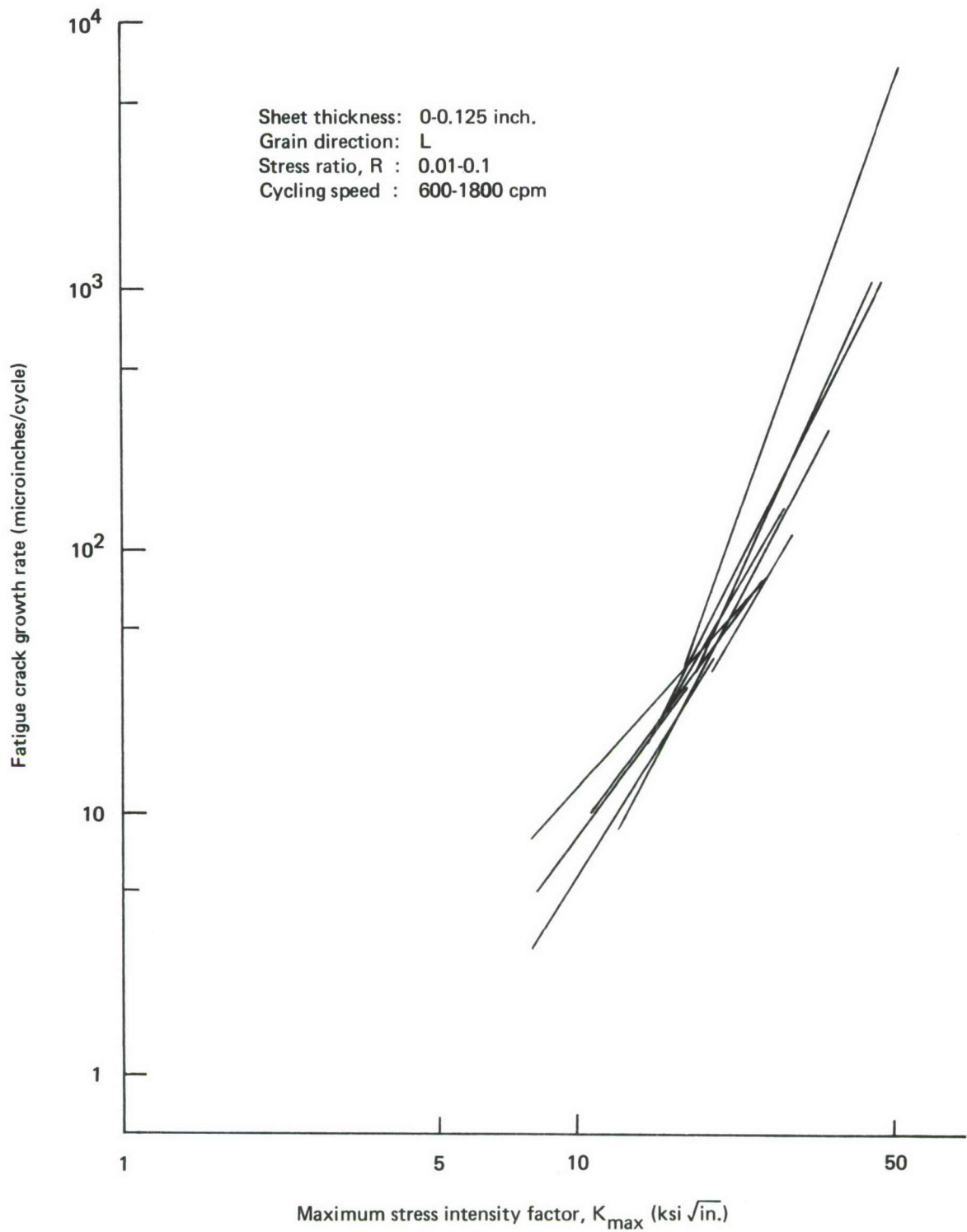
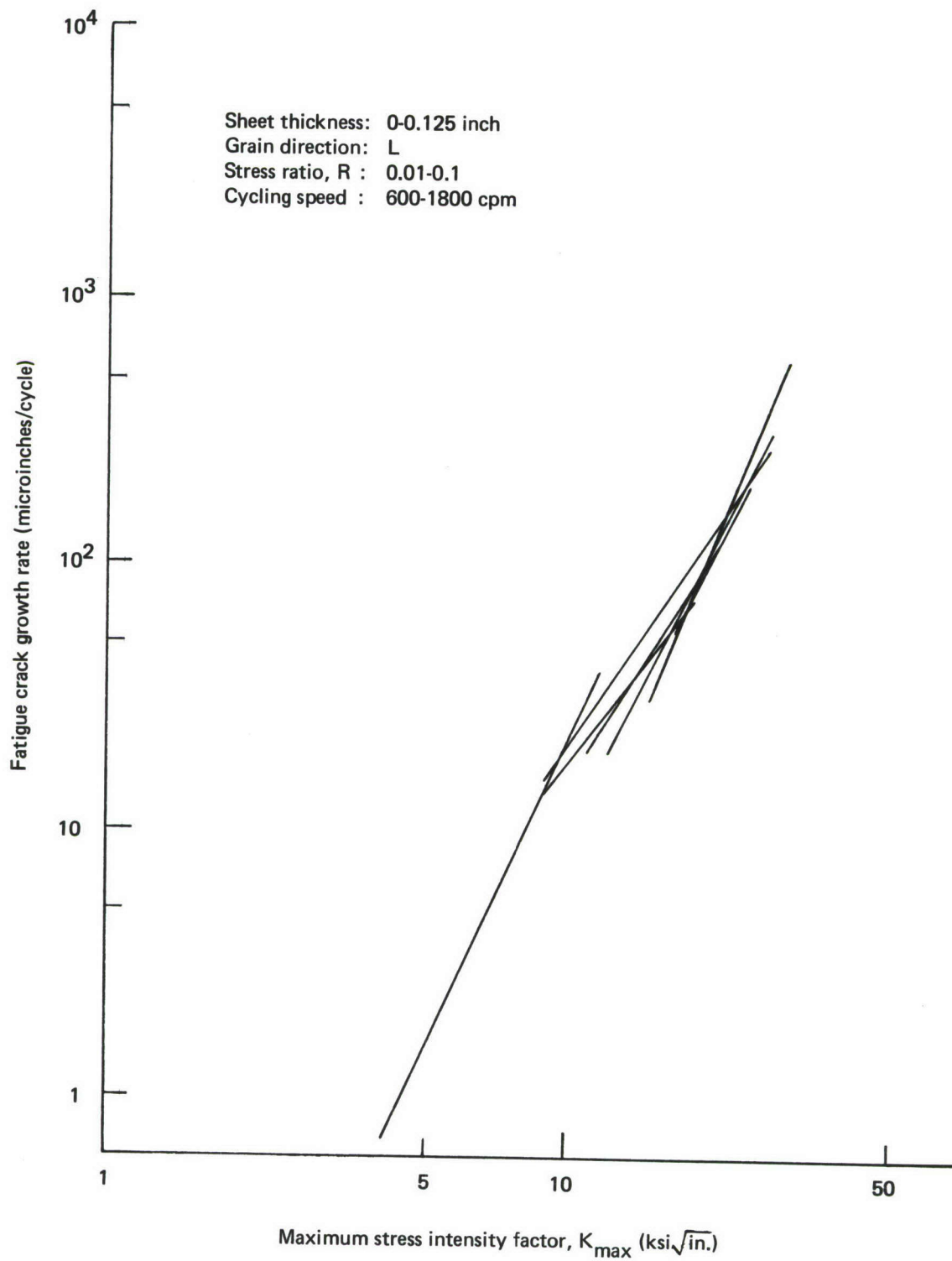


Figure 1.—Fatigue Crack Growth Rate At Low Levels Of  $\Delta K$



*Figure 2.—Fatigue Crack Growth Behavior of 2024-T3 Bare Aluminum Alloy Sheet Tested In Room Air*



*Figure 3.—Fatigue Crack Growth Behavior Of 7075-T6 Bare Aluminum Alloy Sheet Tested In Room Air*



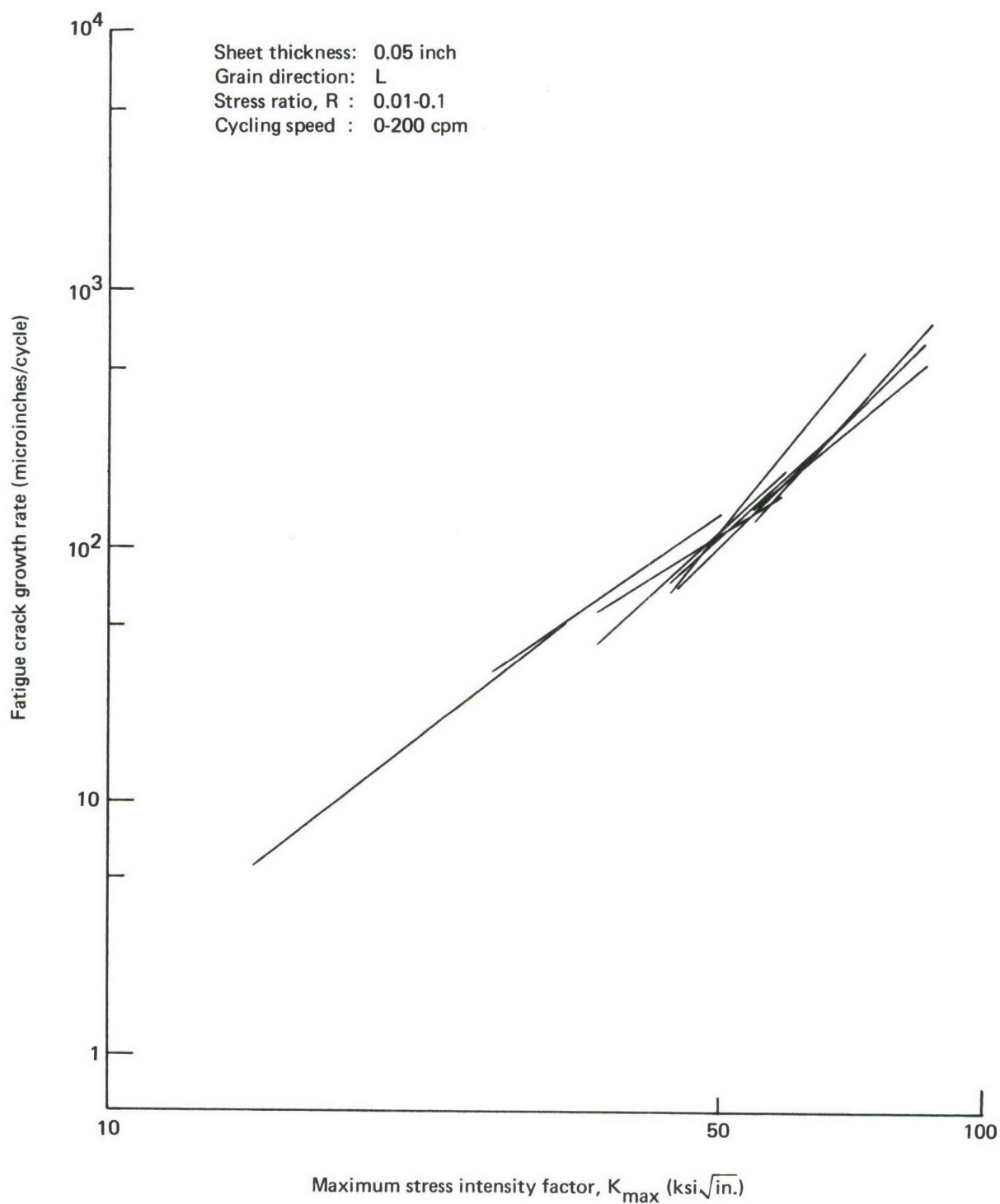


Figure 4.—Fatigue Crack Growth Behavior Of Titanium 8Al-1Mo-1V Duplex Annealed Sheet Tested In Room Air

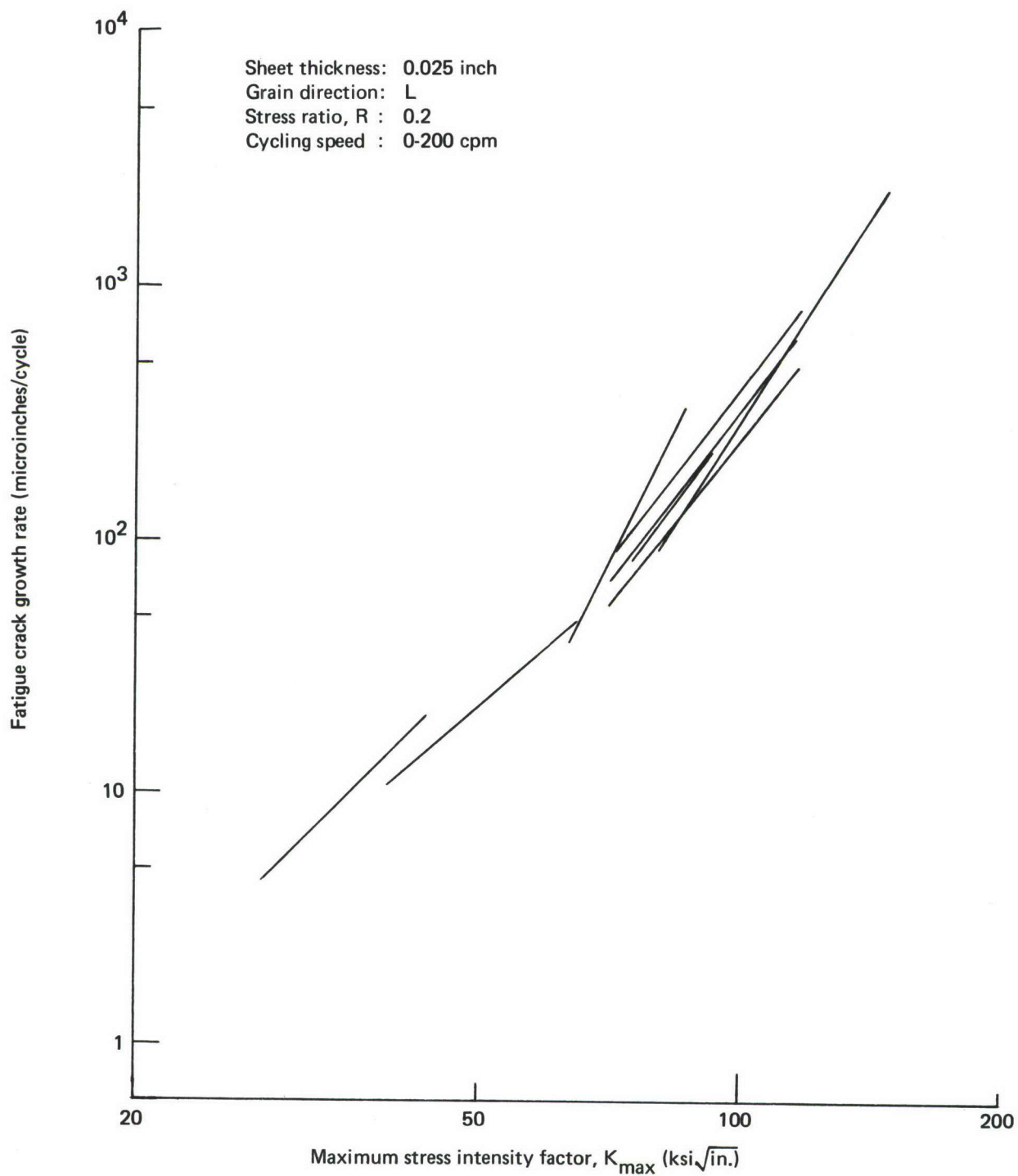


Figure 5.—Fatigue Crack Growth Behavior Of AM350 SCT Steel Sheet Tested In Room Air

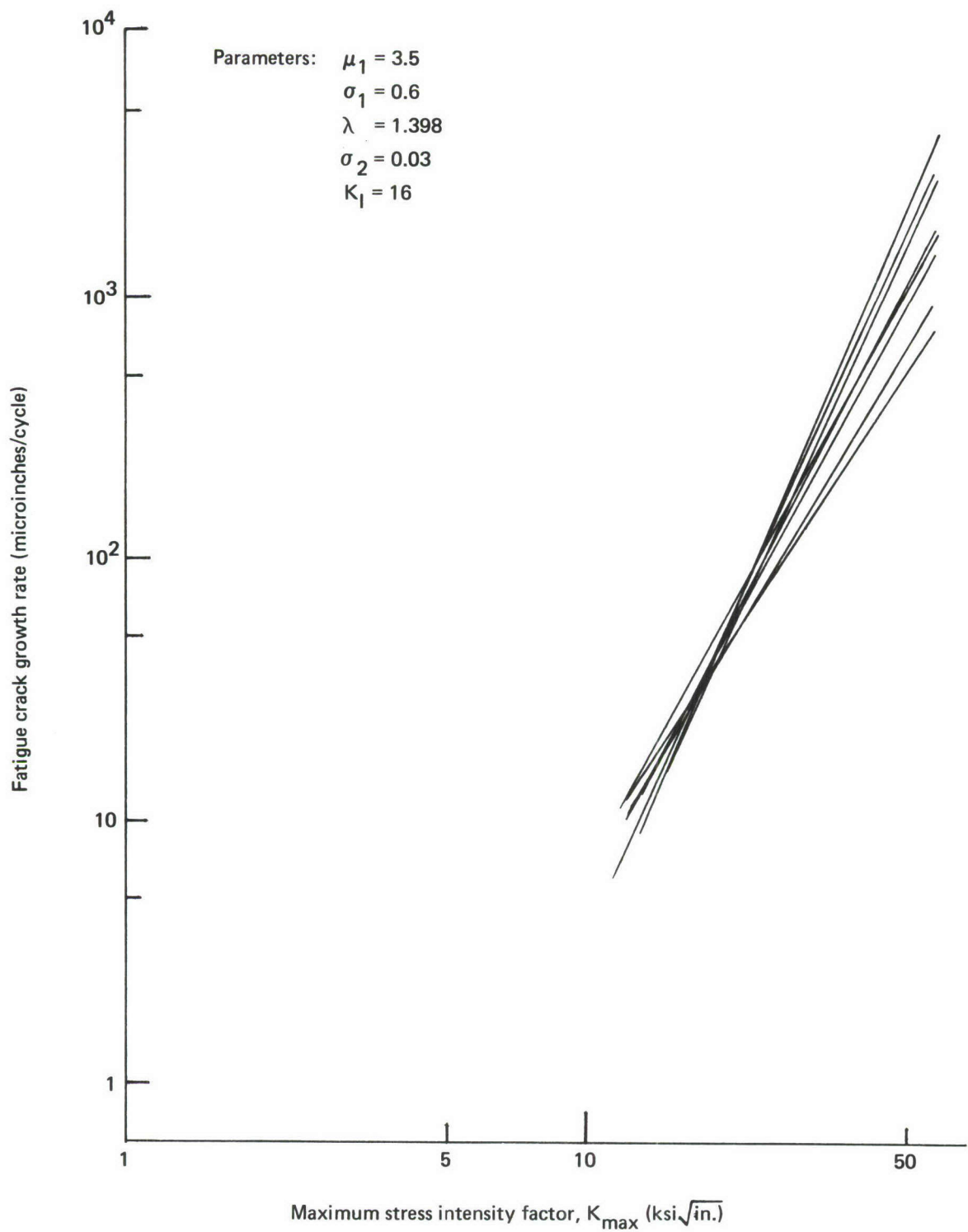


Figure 6.—Typical Random Simulations Of Fatigue Crack Growth Rate Behavior Of 2024-T3 Bare Sheet



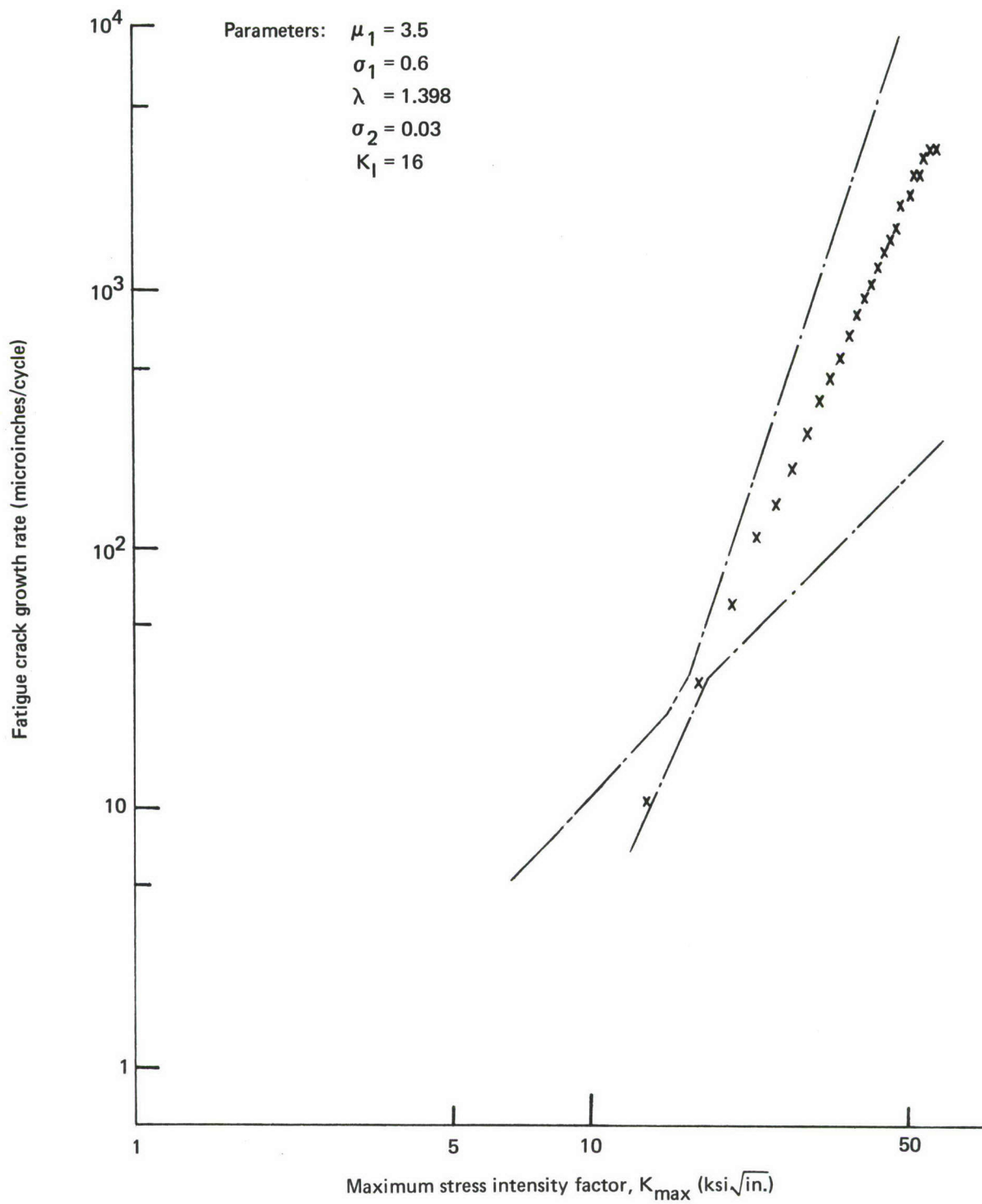


Figure 7.—A Typical Simulation Of 2024-T3 Bare Sheet Crack Growth Rate Behavior And The Extremes From 2000 Such Random Simulations

Parameters:

Shape parameter of crack initiation time,  $\alpha_0 = 4.0$   
 Characteristic life to crack initiation,  $\beta_0 = 120000$  hours  
 Crack length at initiation,  $s_0 = 0.02$  inches  
 Fail-safe length of crack at arrest by a crack stopper,  $s_1 = 14$  inches  
 Fail-safe strength of structure,  $\delta = 0.8$  limit strength

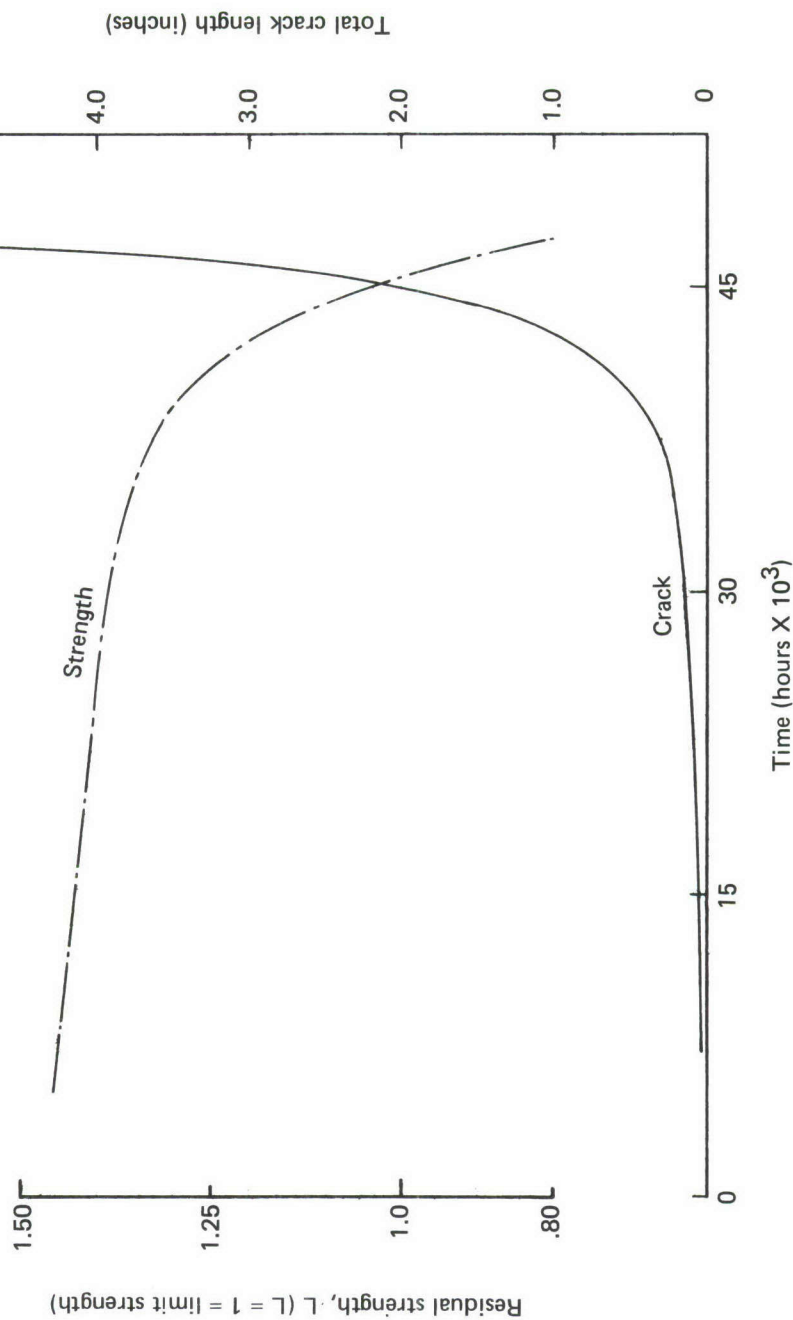


Figure 8.—A Simulated Example Of Crack Growth And Residual Strength Versus Time (One Of The Low Time Cases From 2000 Simulations)

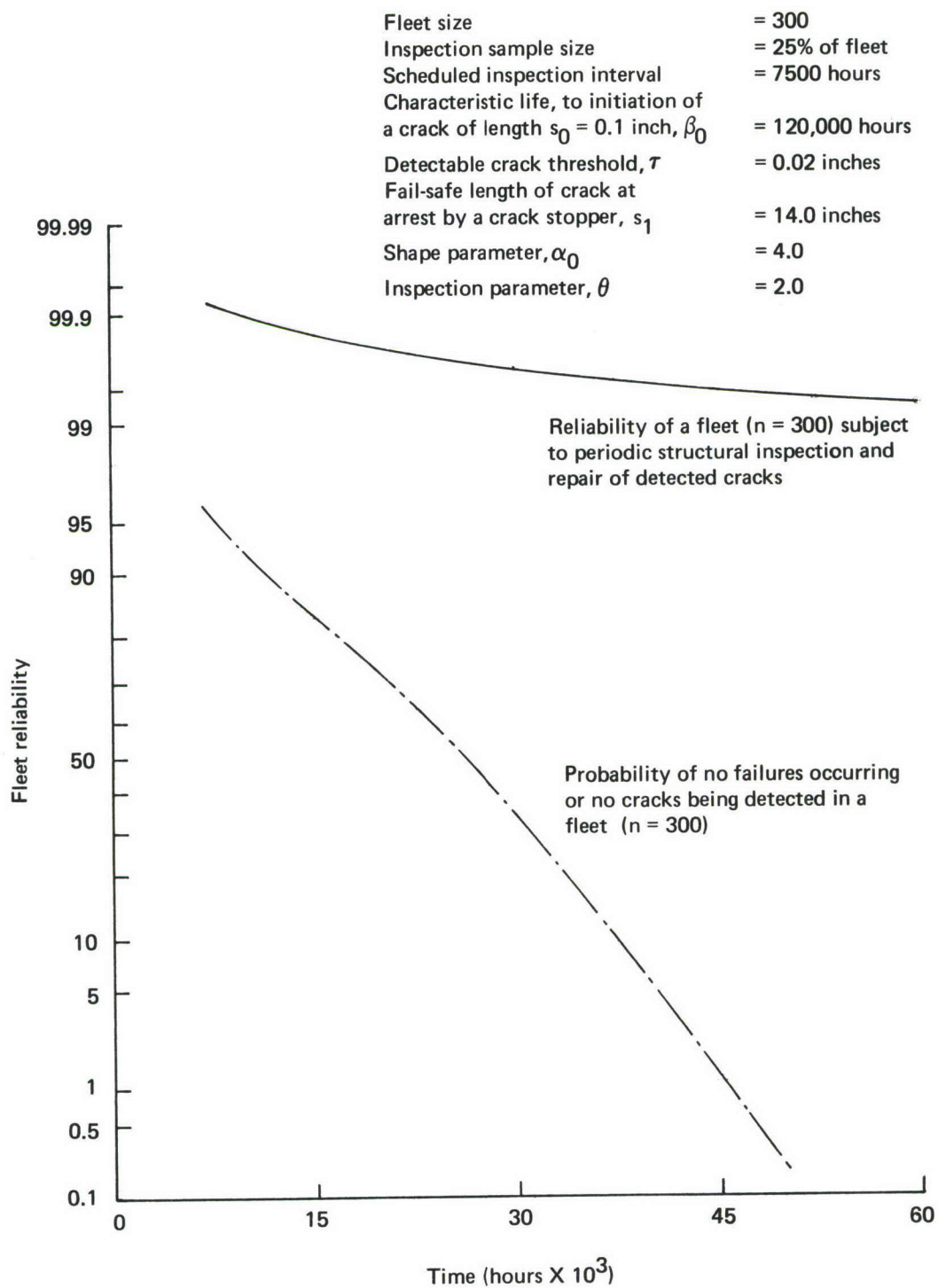


Figure 9.—Influence Of Inspection, And Structural Renewal When Necessary, On Fleet Reliability



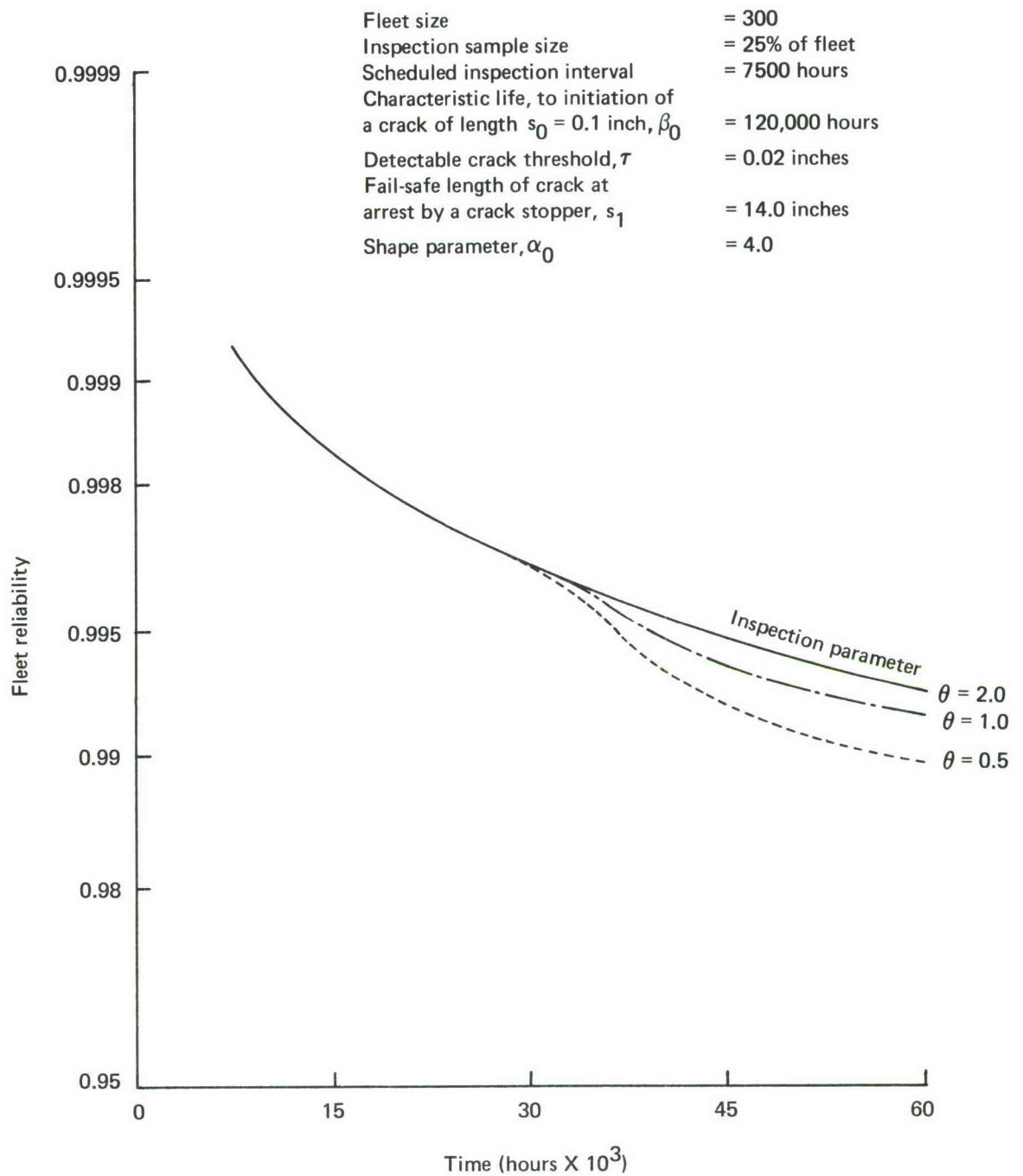


Figure 10.—Influence Of Inspection Parameter On Fleet Reliability, When Initiation Time Shape Parameter Is 4.0

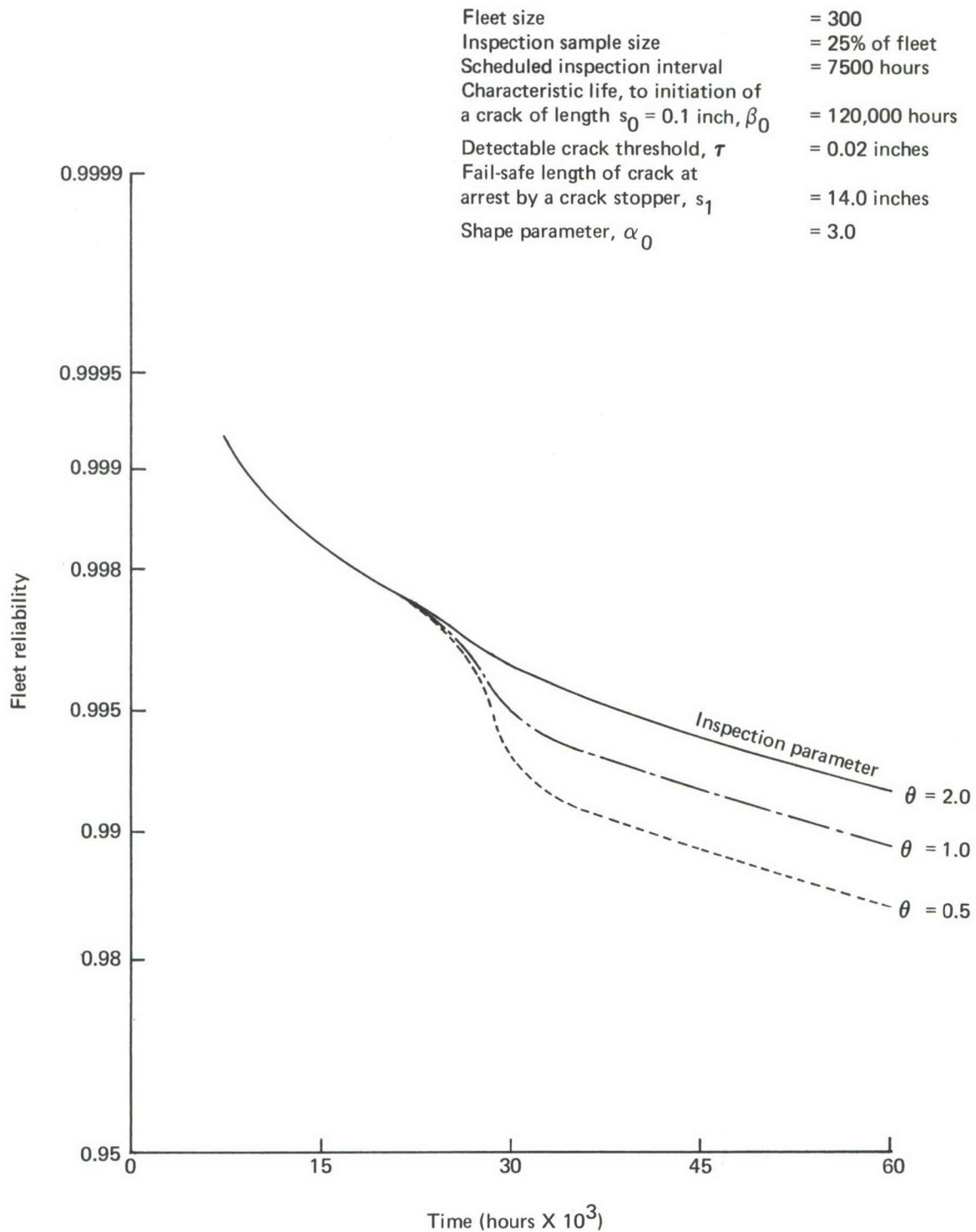
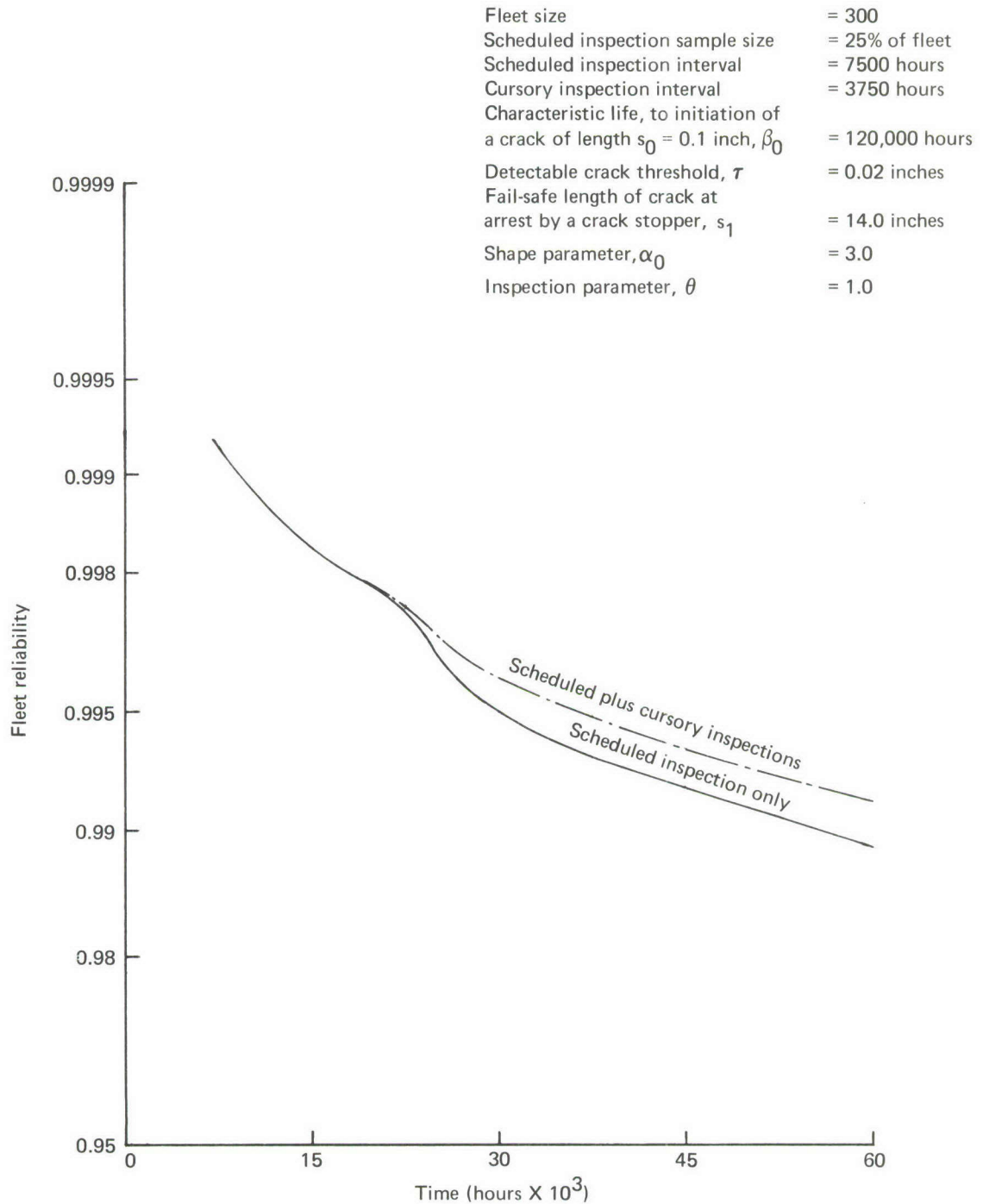


Figure 11.—Influence Of Inspection Parameter On Fleet Reliability, When Initiation Time Shape Parameter Is 3.0



*Figure 12.—Influence Of Additional Cursory Inspections  
On Fleet Reliability*



Fleet size	= 300
Scheduled inspection sample size	= 25% of fleet
Scheduled inspection interval	= 7500 hours
Cursory inspection interval	= 3750 hours
Characteristic life, to initiation of a crack of length $s_0 = 0.1$ inch, $\beta_0$	= 120,000 hours
Detectable crack threshold, $\tau$	= 0.02 inches
Fail-safe length of crack at arrest by a crack stopper, $s_1$	= 14.0 inches
Shape parameter, $\alpha_0$	= 3.0
Inspection parameter, $\theta$	= 1.0

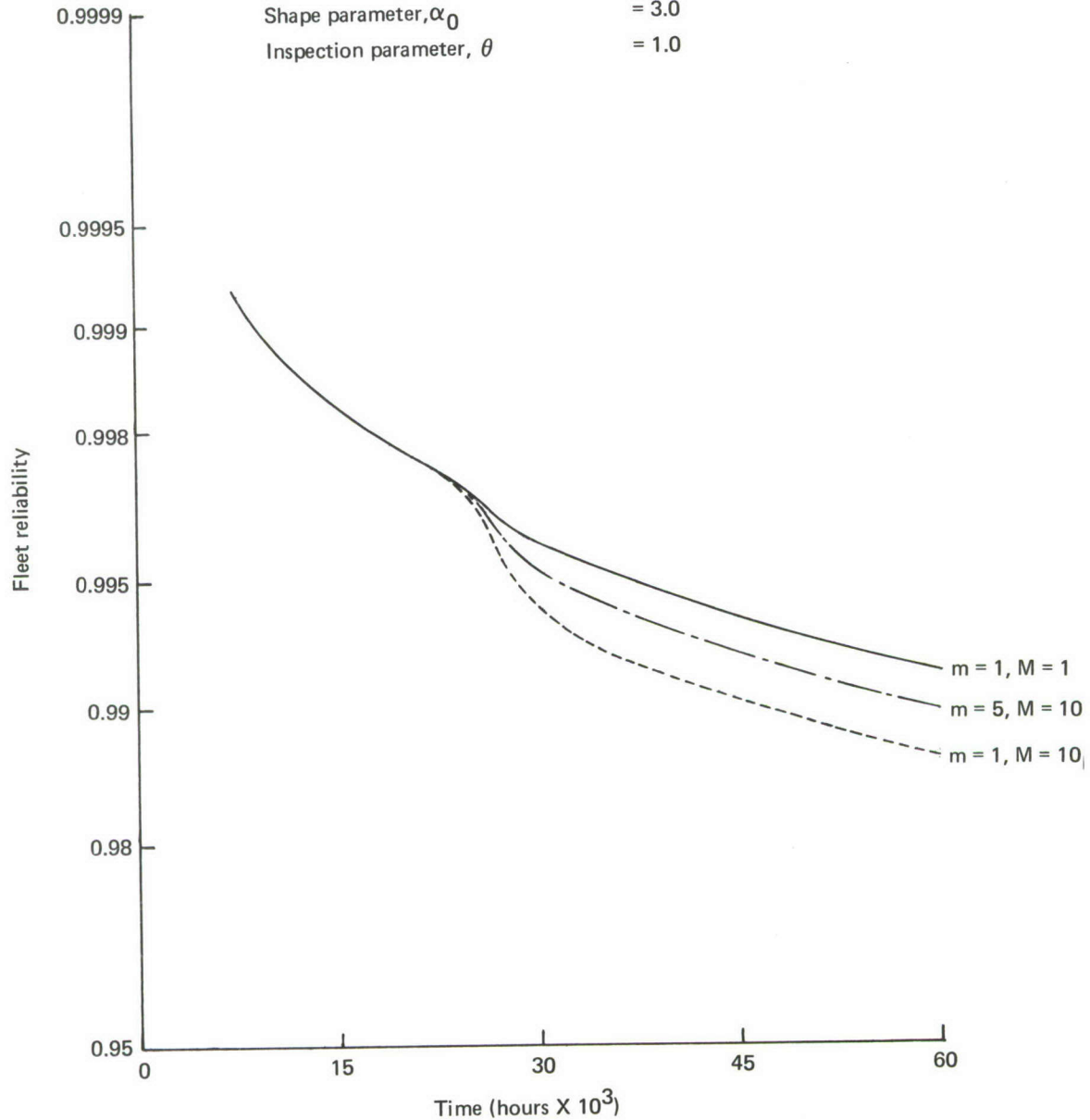


Figure 13.—Influence Of Number Of Details Inspected, From Total In Structure, On Fleet Reliability

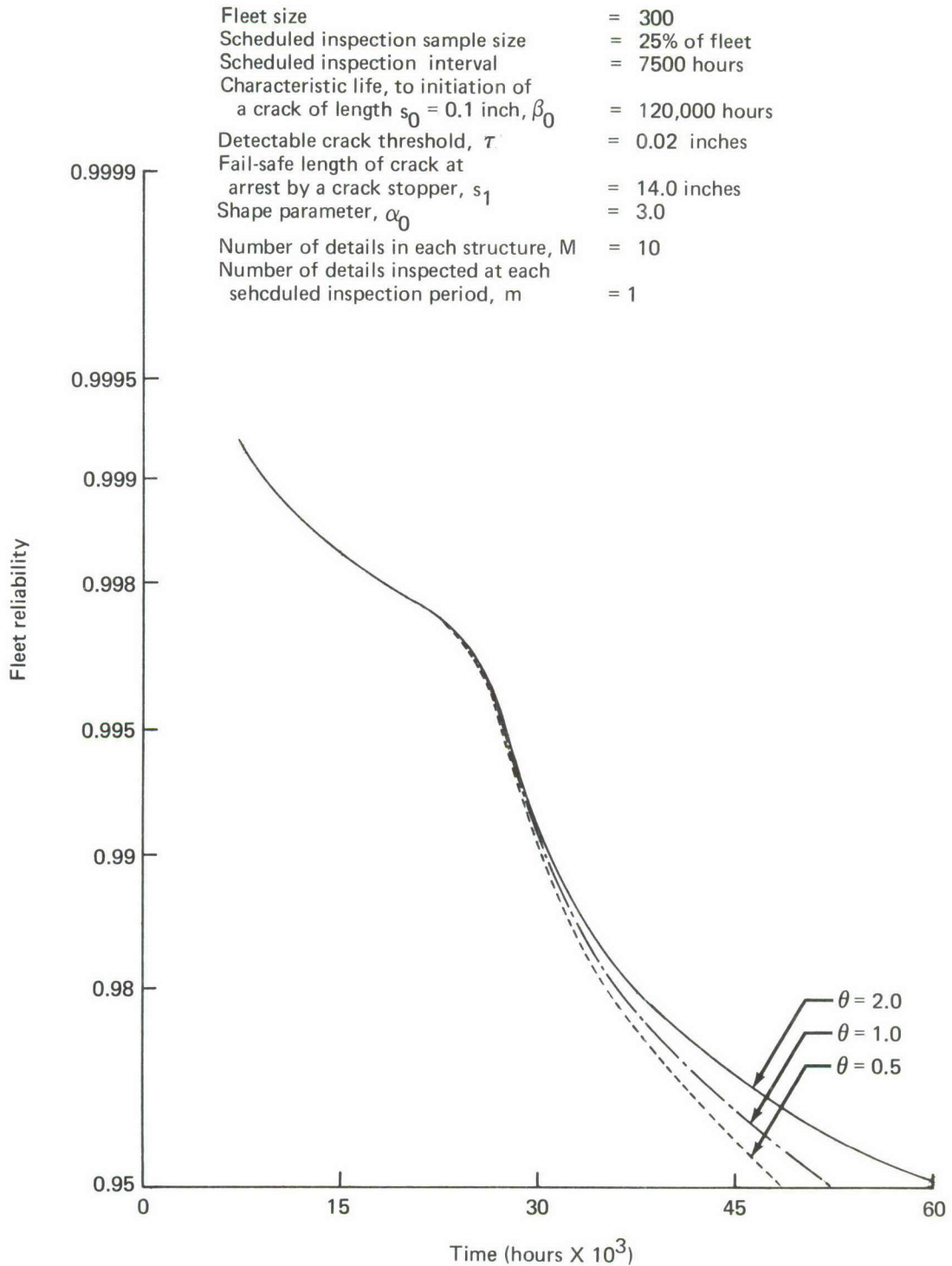


Figure 14.—Influence of Inspection Thoroughness Parameter and Frequency of Cursory Inspection on Fleet Reliability (No Cursory Inspections)

Fleet size	= 300
Scheduled inspection sample size	= 25% of fleet
Scheduled inspection interval	= 7500 hours
Characteristic life, to initiation of a crack of length $s_0 = 0.1$ inch, $\beta_0$	= 120,000 hours
Detectable crack threshold, $\tau$	= 0.02 inches
Fail-safe length of crack at arrest by a crack stopper, $s_1$	= 14.0 inches
Shape parameter, $\alpha_0$	= 3.0
Number of details in each structure, $M$	= 10
Number of details inspected at each scheduled inspection period, $m$	= 1

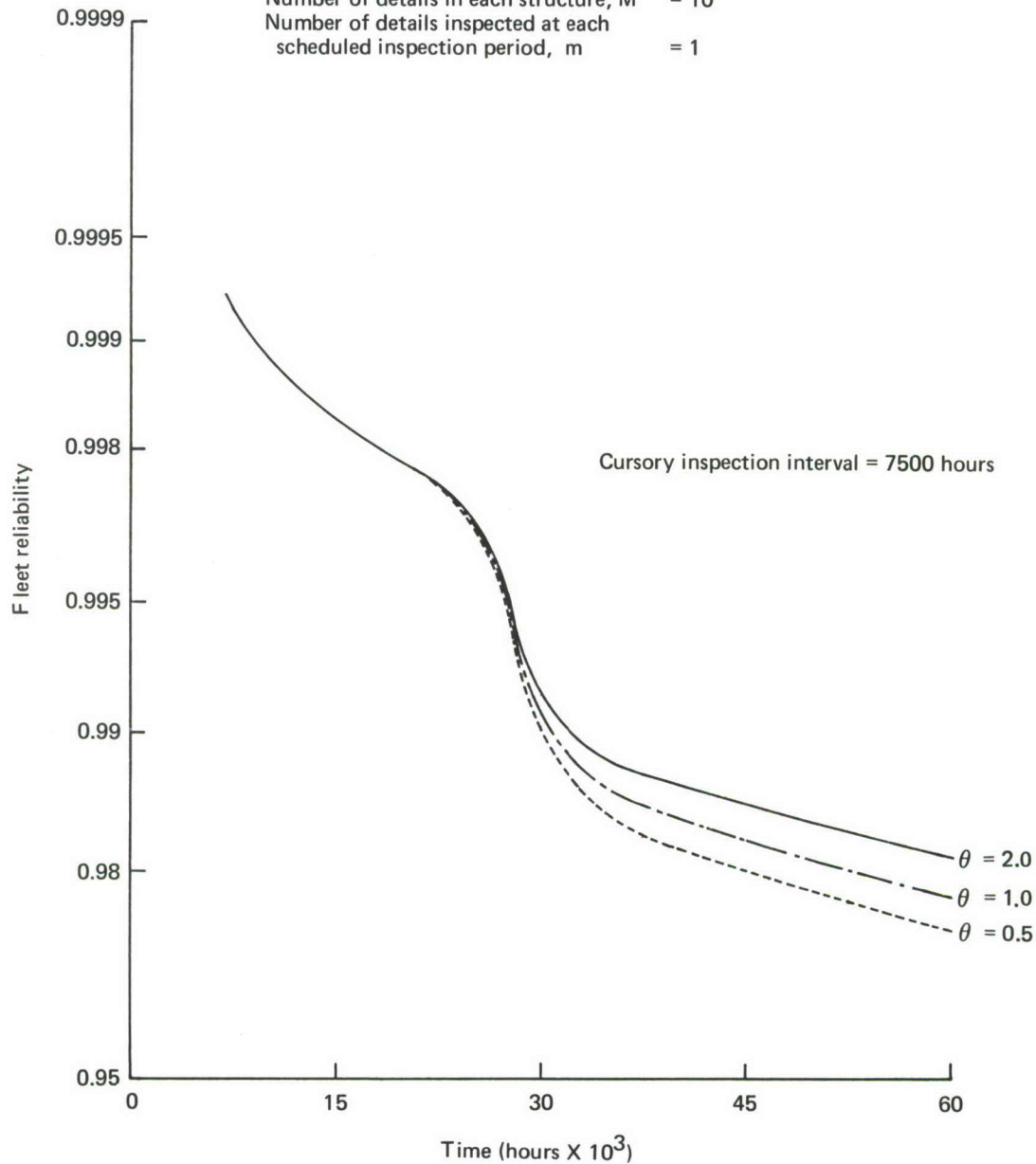


Figure 15.—Influence of Inspection Thoroughness Parameter and Frequency of Cursory Inspection on Fleet Reliability (One Cursory Inspection Per Scheduled Inspection)



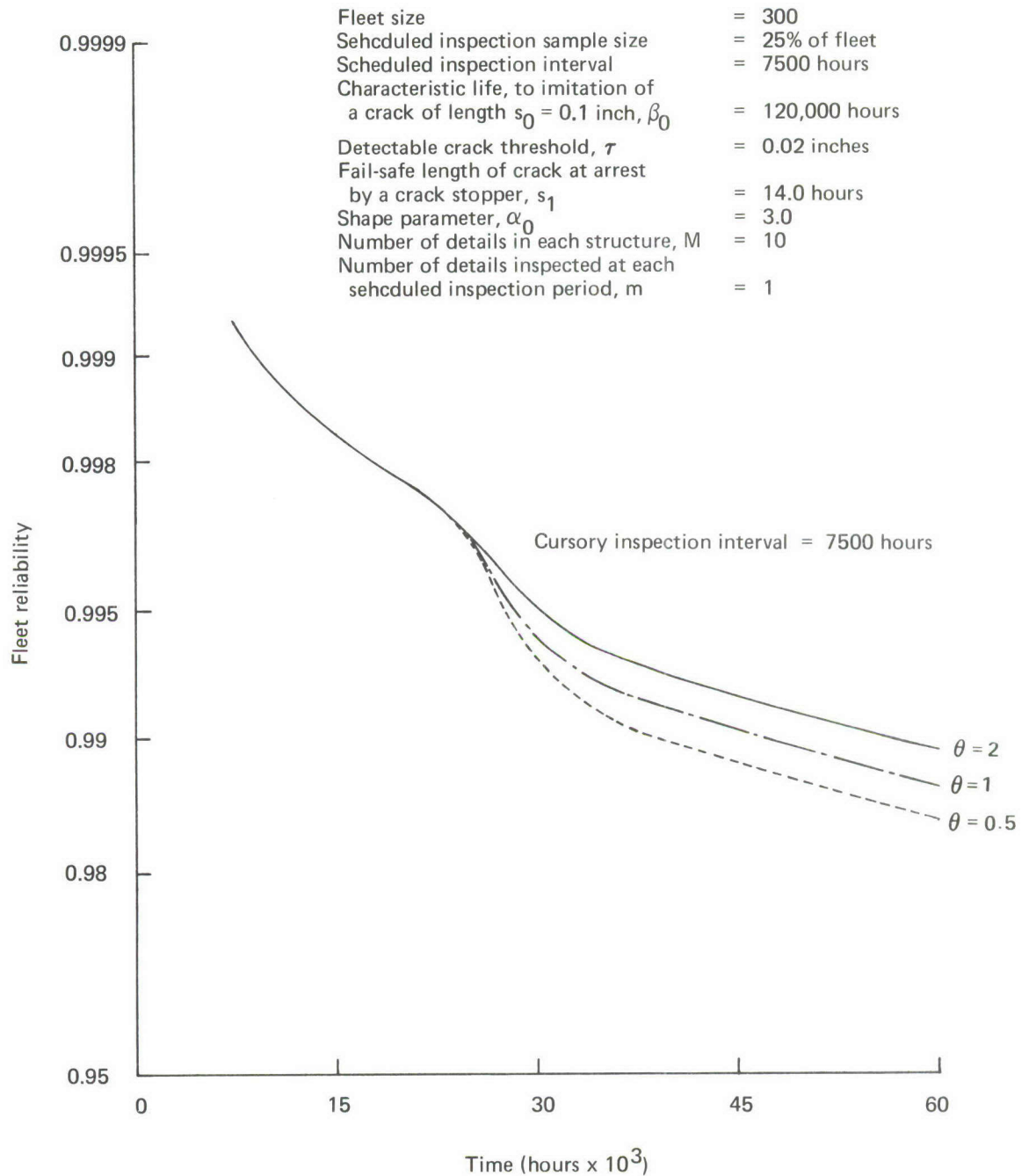


Figure 16.—Influence of Inspection Thoroughness Parameter and Frequency of Cursory Inspection on Fleet Reliability (Two Cursory Inspections Per Scheduled Inspection)

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13. ABSTRACT A method of simulating crack growth has been investigated. The proposed model, which is based on linear elastic fracture mechanics theory, allows for the variability in crack growth behavior found in the experimental data of various materials. Given a reference stress intensity factor range and central tendency values for the crack growth rate and the exponent of the stress intensity factor excursions of a material in a specified configuration, Monte Carlo simulation is used to select various combinations of parameters. These are then used to generate fatigue cracks, on the assumption that crack growth rate is a power function of the stress intensity factor range. The residual strength of the cracked structure is considered to be a decreasing function of the induced crack length. The probability of crack detection also depends on the generated crack and is assumed to improve with increasing crack length. However, this improved detection probability is modified by the probability that the crack location is not the one being inspected. These developments have been used to update the reliability analysis system defined in an earlier report (AFML-TR-72-283). Results of an application of the modified reliability analysis system to a hypothetical, but plausible, situation are presented.			

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14. KEY WORDS	LINK A		LINK B		LINK C	
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Fatigue						
Fail-safe						
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Crack propagation						
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Stress intensity factor						
Residual strength						
Inspection						
Crack detection						
Characteristic life						
Shape parameter						

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